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María Teresa Escobar
Alberto Turón
Ginger Ke
Juan Aguarón

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Multiple Criteria Group Decision Aiding in Seven-valued Logic

Salvatore Greco¹ and Roman Słowiński²

¹ Department of Economics and Business, University of Catania,
Corso Italia, 55, 95129 Catania, Italy

² Institute of Computing Science, Poznań University of Technology,
60-965 Poznań, and Systems Research Institute,
Polish Academy of Sciences, 01-447 Warsaw, Poland

Abstract. We are considering a group decision problem concerning the ranking of a set of alternatives evaluated by a set of criteria. The evaluations of alternatives on individual criteria are in conflict, as achieving a better score on one criterion necessitates accepting a deterioration on at least one other criterion. Moreover, diverse experts, referred to as decision-makers, generally exhibit differing preferences across the criteria, making the decision problem doubly challenging, as it requires both a group and a multi-criteria compromise. To model preferences in this complex problem, we are using the seven-valued logic proposed recently by the authors for Multiple Criteria Decision Aiding (MCDA). We assume that the decision-makers, use a weighted aggregation of multi-criteria evaluations of alternatives. Since the individual preferences of decision-makers are not fixed with certainty to specific weight vectors, we consider, for each expert, a cloud of possible criteria weights resulting from a perturbation of their central weight vectors. This approach not only accounts for the uncertainty in experts' preferences but also increases the likelihood of finding a compromise ranking of alternatives within the cumulated clouds of weights. Our approach effectively addresses common challenges in preference modeling for group decision and MCDA, such as uncertainty, imprecision, and ill-determination of performances and preferences. To this end, we present a specific procedure to construct a seven-valued preference relation and use it to define the ranking recommendation that considers robustness concerns. We illustrate and discuss the application of our approach using a didactic example. We also propose directions for future development of the proposed methodology.

Keywords: Group decision; Multiple criteria decision aiding; Preference representation; Seven-valued logic; Robustness concern

1 Introduction

The seven-valued logic considered in this paper has been recently introduced by the authors in the context of rough-set-based reasoning about data [5] in order

to represent uncertainty following from data granularity. Later, we adapted this logic to represent preferences in Multiple Criteria Decision Aiding (MCDA) [7].

To explain how seven-valued logic can be helpful in group decision-making concerning the ranking of a set of alternatives evaluated on multiple criteria, let us consider the following example. Consider a hypothetical problem of group ranking for a set \mathcal{A} of possible factory locations with respect to sustainable development. Suppose that three macro-criteria are used to evaluate the locations: economic (Eco), social (Soc), and environmental (Env). Three experts, representing diverse perspectives on evaluating locations, serve as the decision-makers. Assume, moreover, that each decision-maker (DM) evaluates each location $a \in \mathcal{A}$ using a weighted sum denoted by $U(a)$:

$$U(a) = w_{Eco} \times Eco(a) + w_{Soc} \times Soc(a) + w_{Env} \times Env(a), \quad w_{Eco} + w_{Soc} + w_{Env} = 1,$$

and $w_{Eco} \geq 0$, $w_{Soc} \geq 0$, $w_{Env} \geq 0$. The differing perspectives of the experts are reflected in the various weights they assign to the macro-criteria:

- DM_{Eco} shows an *economic perspective*, with $w_{Eco} > w_{Soc} = w_{Env}$,
- DM_{Soc} shows a *social perspective*, with $w_{Soc} > w_{Eco} = w_{Env}$,
- DM_{Env} shows an *environmental perspective*, with $w_{Env} > w_{Eco} = w_{Soc}$.

It happens, however, that DMs identified with particular perspectives do not have strongly established individual preferences which could be expressed as single vectors of weights. Instead, the DMs agree to provide some central weight vectors satisfying the above constraints. To make the preference representation more robust, for each DM, a cloud of possible criteria weights is generated by perturbation of the corresponding central weight vector. For example, if the central weight vector of DM_{Eco} is such that weight w_{Eco} is set at 0.5, the other weights, w_{Soc} and w_{Env} , are each set to 0.25. The cloud is obtained by perturbation of the central weight vector within a given range of $r\%$, with a simultaneous adjustment of weights ensuring that their sum equals always 1. Therefore, instead of a single overall evaluation by each DM, each location $a \in \mathcal{A}$ gets a set of overall evaluations from each DM - including the central evaluation and a series of evaluations resulting from the perturbation. Let us denote by $\mathcal{U}^{Eco}(a)$, $\mathcal{U}^{Soc}(a)$, and $\mathcal{U}^{Env}(a)$ the sets of overall evaluations of $a \in \mathcal{A}$ by the three DMs, respectively.

Evaluations related to one DM will be denoted by \mathcal{U}^p , where p can be Eco , Soc , or Env . Comparing location a with location b ($a, b \in \mathcal{A}$) using evaluations \mathcal{U}^p of DM_p , there are three possible situations:

- a is at least as good as b , because a is at least as good as b taking the central weight vector of DM_p as well as all its ‘perturbations’, that is, $U(a) \geq U(b)$ for all $U \in \mathcal{U}^p$,
- a is not at least as good as b , because a is worse than b taking the central weight vector of DM_p as well as all its ‘perturbations’, that is, $U(a) < U(b)$ for all $U \in \mathcal{U}^p$,
- it is unknown whether a is at least as good as b , because a is at least as good as b for some evaluations of DM_p but worse for others, that is, $U(a) \geq U(b)$ for some $U \in \mathcal{U}^p$ and $U(a) < U(b)$ for some other $U \in \mathcal{U}^p$.

In result of the pairwise comparisons of location a and location b across the entire set of overall evaluations by all three DMs, the proposition “location a is at least as good as location b ”, denoted by $a \succsim b$, can assume one of the following seven possible states of truth:

- a is at least as good as b for all three DMs, that is, a is at least as good as b for all the evaluations of all three DMs; then, proposition $a \succsim b$ is **true**;
- a is at least as good as b for one or two DMs, and it is unknown for the other DMs, that is, a is at least as good as b for all the evaluations of one or two DMs, but there are evaluations of the remaining DMs for which the comparison gives mixed results: sometimes a is at least as good as b and sometimes not; then, proposition $a \succsim b$ is **sometimes true**;
- it is unknown whether a is at least as good as b for all three DMs, that is, for each DM there are evaluations for which a is at least as good as b and others for which this is not true; then, proposition $a \succsim b$ is **unknown**;
- a is at least as good as b for one or two DMs and this is false for the other DMs, that is, a is at least as good as b for all the evaluations of one or two DMs while this is false for all the evaluations of the other DMs; then, proposition $a \succsim b$ is **contradictory**;
- a is at least as good as b for one DM, it is false for another DM, and it is unknown for the remaining DM, that is, a is at least as good as b for all the evaluations of one DM, it is false for all the evaluations of another DM, and it is true for some evaluations and false for other evaluations of the remaining DM; then, proposition $a \succsim b$ is **fully contradictory**;
- a is not at least as good as b for one or two DMs and it is unknown for the other DMs, that is, a is not at least as good as b for all the evaluations of one or two DMs, but there are evaluations of the remaining DMs for which the comparison gives mixed results: sometimes a is at least as good as b and sometimes not; then, proposition $a \succsim b$ is **sometimes false**;
- a is not at least as good as b for all three DMs, that is, a is not at least as good as b for all the evaluations of all three DMs; then, proposition $a \succsim b$ is **false**.

The lattice presented in Figure 1 illustrates the graded values of truth. It is represented in the system of coordinates: ‘certainty of truth’ vs. ‘diversity of arguments’.

In this paper, we take advantage of the seven-valued logic to handle robustness concerns in group decision concerning multi-criteria ranking of alternatives. In the next Section, we sketch the presented methodology using a block scheme representing its main steps. In Section 3, we explain the methodology with a didactic example. The last section groups conclusions.

2 Block scheme of the proposed methodology

The block scheme summarizing the proposed methodology is presented in Figure 2. For each DM, the simplex of weight vectors is obtained by a perturbation of the DM’s central weight vector within the range of $r\%$.

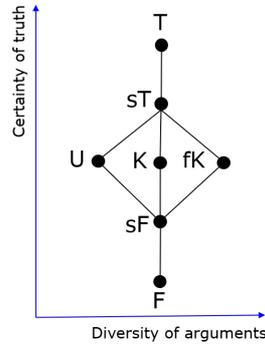


Fig. 1. The lattice of truth of the seven-valued logic.

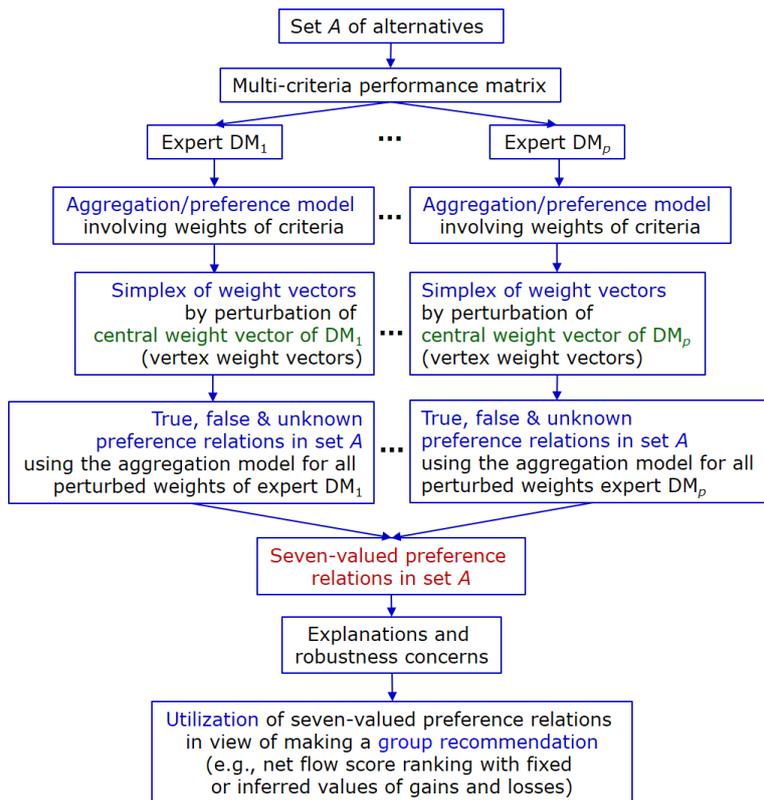


Fig. 2. The block scheme of the basic methodology.

3 Illustration of the methodology on a didactic example

3.1 Group ranking of students

In this section, we are explaining the methodology of group decision based on seven-valued representation of preferences using a simple example. Consider a group of three professors who want to rank five students, taking into account their grades in Mathematics (*Math*), Physics (*Phys*), Literature (*Lit*), and Philosophy (*Phil*). These grades, expressed on a scale from 0 to 100, are presented in Table 1 [7].

Table 1. Grades of five students in Mathematics, Physics, Literature and Philosophy.

Student\Subject	Mathematics	Physics	Literature	Philosophy
S1	80	90	50	70
S2	70	80	80	70
S3	100	60	50	70
S4	90	90	60	60
S5	80	80	70	70

Suppose the professors agree to compare students using a value function $U : [0, 100]^4 \rightarrow [0, 100]$ assigning to each student S the overall evaluation

$$U(S) = w_{Math} \times Math(S) + w_{Phys} \times Phys(S) + w_{Lit} \times Lit(S) + w_{Phil} \times Phil(S)$$

with

- $Math(S), Phys(S), Lit(S)$ and $Phil(S)$ being the grades of student S in Mathematics, Physics, Literature and Philosophy, respectively,
- $w_{Math}, w_{Phys}, w_{Lit}, w_{Phil}$, such that $w_{Math} \geq 0, w_{Phys} \geq 0, w_{Lit} \geq 0, w_{Phil} \geq 0, w_{Math} + w_{Phys} + w_{Lit} + w_{Phil} = 1$, being the weights of Mathematics, Physics, Literature and Philosophy, respectively.

In this case, the weights $w_{Math}, w_{Phys}, w_{Lit}$ and w_{Phil} and, more precisely, their ratios, represent the trade-offs between the grades of four subjects. A ratio w_i/w_j says how many points of subject j compensate one point of subject i in the value function. These weights can be determined using a procedure coherent with their meaning, such as SMART or SMARTER [1]. Using the value function for comparing any two students S, S' , we conclude that S is at least as good as S' if $U(S) \geq U(S')$.

The professors participating in the ranking of students represent diverse perspectives on overall evaluations:

- DM_1 represents an *egalitarian* perspective with respect to Sciences and Humanities, that is, Mathematics and Physics on one hand, and Literature and Philosophy on the other hand, so that equal weights are assigned by this DM to all the four subjects in the central weight vector: $w_{Math}^1 = w_{Phys}^1 = w_{Lit}^1 = w_{Phil}^1 = 0.25$;

- DM₂ represents an *extreme* perspective which gives a strong advantage to Sciences over Humanities, so that Mathematics and Physics are getting much larger weights than Literature and Philosophy in the central weight vector of this DM: $w_{Math}^2 = w_{Phys}^2 = 0.4$ and $w_{Lit}^2 = w_{Phil}^2 = 0.1$;
- DM₃ represents a *moderate* perspective, intermediate between the egalitarian and extreme perspectives, which gives a slight advantage to Sciences over Humanities, so that Mathematics and Physics are getting a bit larger weights than Literature and Philosophy in the central weight vector of this DM: $w_{Math}^3 = w_{Phys}^3 = 0.3$ and $w_{Lit}^3 = w_{Phil}^3 = 0.2$.

The overall evaluations of the five students by professors representing the three perspectives are presented in Table 2.

Table 2. Overall evaluations of five students by value functions of three professors.

Student\Professor	DM ₁	DM ₂	DM ₃
S1	72.5	80	75
S2	75	75	75
S3	70	76	72
S4	75	84	78
S5	75	78	76

Looking at Table 2, one can note that

- S1 has a better evaluation than S3 by all three professors,
- S4 has a not worse evaluation than all other students by all three professors,
- S5 has a not worse evaluation than S2 and S3 by all three professors,
- for all other pairs of students there is no definite preference by all three professors, because for each pair S, S' , student S is better than S' for some professor, and student S' is better than S for another professor.

3.2 Construction of the seven-valued preference relations with value function aggregation

To address the robustness concerns let us check how overall evaluations might change if the central weight vectors of professors were perturbed within the range of, say 15%. Consequently, for each professor, the perturbed weight vectors

$$\tilde{\mathbf{w}}^p = [\tilde{w}_{Math}^p, \tilde{w}_{Phys}^p, \tilde{w}_{Lit}^p, \tilde{w}_{Phil}^p],$$

$p = 1, 2, 3$, satisfying the following set of constraints, are considered [7]:

$$\left. \begin{aligned} &\tilde{w}_{Math}^p \geq 0, \tilde{w}_{Phys}^p \geq 0, \tilde{w}_{Lit}^p \geq 0, \tilde{w}_{Phil}^p \geq 0, \\ &\tilde{w}_{Math}^p + \tilde{w}_{Phys}^p + \tilde{w}_{Lit}^p + \tilde{w}_{Phil}^p = 1, \\ &w_{Math}^p(1-r) \leq \tilde{w}_{Math}^p \leq w_{Math}^p(1+r), \\ &w_{Phys}^p(1-r) \leq \tilde{w}_{Phys}^p \leq w_{Phys}^p(1+r), \\ &w_{Lit}^p(1-r) \leq \tilde{w}_{Lit}^p \leq w_{Lit}^p(1+r), \\ &w_{Phil}^p(1-r) \leq \tilde{w}_{Phil}^p \leq w_{Phil}^p(1+r). \end{aligned} \right\} E_{(wp)}^p$$

The overall evaluation of student S by the value function with weight vector $\tilde{\mathbf{w}}^p$ of DM_p is denoted by $U(S, \tilde{\mathbf{w}}^p)$, $p = 1, 2, 3$, that is:

$$U(S, \tilde{\mathbf{w}}^p) = \tilde{w}_{Math}^p \times Math(S) + \tilde{w}_{Phys}^p \times Phys(S) + \tilde{w}_{Lit}^p \times Lit(S) + \tilde{w}_{Phil}^p \times Phil(S).$$

Taking into account the perturbed weights of one DM_p , $p \in \{1, 2, 3\}$, we conclude that the proposition “student S is at least as good as student S' for DM_p ” is:

- true, and denoted by $S \succsim^{p,T} S'$, if $U(S, \tilde{\mathbf{w}}^p) \geq U(S', \tilde{\mathbf{w}}^p)$ for all $\tilde{\mathbf{w}}^p$ satisfying the constraints $E_{(wp)}^p$;
- false, and denoted by $S \not\sucsim^{p,F} S'$, if $U(S, \tilde{\mathbf{w}}^p) < U(S', \tilde{\mathbf{w}}^p)$ for all $\tilde{\mathbf{w}}^p$ satisfying the constraints $E_{(wp)}^p$;
- unknown, and denoted by $S \not\sucsim^{p,U} S'$, if $U(S, \tilde{\mathbf{w}}^p) \geq U(S', \tilde{\mathbf{w}}^p)$ for some $\tilde{\mathbf{w}}^p$ satisfying the constraints $E_{(wp)}^p$ and $U(S, \tilde{\mathbf{w}}^p) < U(S', \tilde{\mathbf{w}}^p)$ for some other $\tilde{\mathbf{w}}^p$ satisfying the same constraints.

Taking into account the perturbed weights of all three professors DM_p , $p = 1, 2, 3$, we conclude that the proposition “student S is at least as good as student S' for all professors” is:

- **true**, and denoted by $S \succsim^T S'$, if $S \succsim^{p,T} S'$ for all DM_p , $p = 1, 2, 3$;
- **sometimes true**, and denoted by $S \succsim^{sT} S'$, if $S \succsim^{p,T} S'$ for one or two DM_p , $p \in \{1, 2, 3\}$, and $S \not\sucsim^{r,U} S'$ for other DM_r , $r \in \{1, 2, 3\}$, with $r \neq p$;
- **unknown**, and denoted by $S \not\sucsim^U S'$, if $S \not\sucsim^{p,U} S'$ for all DM_p , $p = 1, 2, 3$;
- **contradictory**, and denoted by $S \not\sucsim^K S'$, if $S \succsim^{p,T} S'$ for one or two DM_p , $p \in \{1, 2, 3\}$, and $S \not\sucsim^{r,F} S'$ for other DM_r , $r \in \{1, 2, 3\}$, with $r \neq p$;
- **fully contradictory**, and denoted by $S \not\sucsim^{fK} S'$, if $S \succsim^{p,T} S'$ for one DM_p , $p \in \{1, 2, 3\}$, $S \not\sucsim^{r,F} S'$ for another DM_r , $r \in \{1, 2, 3\}$, and $S \not\sucsim^{s,U} S'$ for the remaining DM_s , $s \in \{1, 2, 3\}$, with $s \neq r \neq p$;
- **sometimes false**, and denoted by $S \not\sucsim^{sF} S'$, if $S \not\sucsim^{p,F} S'$ for one or two DM_p , $p \in \{1, 2, 3\}$, and $S \not\sucsim^{r,U} S'$ for other DM_r , $r \in \{1, 2, 3\}$, with $r \neq p$;
- **false**, and denoted by $S \not\sucsim^F S'$, if $S \not\sucsim^{p,F} S'$ for all DM_p , $p = 1, 2, 3$.

Clearly, $E_{(wp)}^p$ is a convex polyhedron (simplex) in \mathbb{R}^4 and the points of $E_{(wp)}^p$ are all and only the convex combinations of its vertices. More precisely, denoting the set of vertices of $E_{(wp)}^p$ by $V(E_{(wp)}^p)$, for all $\tilde{\mathbf{w}}^p \in E_{(wp)}^p$, we have [7]:

$$\tilde{\mathbf{w}}^p = \sum_{\hat{\mathbf{w}}^p \in V(E_{(wp)}^p)} \alpha_{\hat{\mathbf{w}}^p} \times \hat{\mathbf{w}}^p$$

with $\alpha_{\hat{\mathbf{w}}^p} \geq 0$ for all vertices $\hat{\mathbf{w}}^p \in V(E_{(wp)}^p)$ and $\sum_{\hat{\mathbf{w}}^p \in V(E_{(wp)}^p)} \alpha_{\hat{\mathbf{w}}^p} = 1$.

To compute the preference relations $\succsim^{p,R}$, $R \in \{T, F, U\}$, for each particular DM_p , $p \in \{1, 2, 3\}$, and, on this basis, the overall seven-valued preference relations \succsim^H , $H \in \{T, sT, U, K, fK, sF, F\}$ for the group of professors, we will use the following proposition proved in [7].

Proposition. *For all pairs of students, S and S' , and constraints $E_{(wp)}^p$ on perturbed central weight vectors of decision-makers DM_p , $p \in \{1, 2, 3\}$, it holds:*

$S \succsim^{\lambda^{p,T}} S'$ if and only if $m^p(S, S') \geq 0$,
 $S \succsim^{\lambda^{p,F}} S'$ if and only if $M^p(S, S') < 0$,
 $S \succsim^{\lambda^{p,U}} S'$, if and only if $m^p(S, S') < 0 \leq M^p(S, S')$,
 where
 $m^p(S, S') = \min[U(S) - U(S')] \text{ subject to } E_{(wp)}^p$,
 $M^p(S, S') = \max[U(S) - U(S')] \text{ subject to } E_{(wp)}^p$.

In Tables 3, 4 and 5, we present the results of the application of the Proposition, i.e., the values of $m^p(S, S')$ and $M^p(S, S')$, and the resulting preference relations $\succsim^{p,R}$, $R \in \{T, F, U\}$, for each particular decision maker DM_p , $p \in \{1, 2, 3\}$, respectively.

Table 3. Values of $m^1(S, S')$ and $M^1(S, S')$ (in parenthesis), and resulting preference relations between students for the egalitarian DM_1 and value function aggregation: $\succsim^{1,T}$, $\succsim^{1,F}$, and $\succsim^{1,U}$

Student	S1	S2	S3	S4	S5
S1	(0,0) $\rightarrow_{\succsim^{1,T}}$	(-4.375, -0.625) $\rightarrow_{\succsim^{1,F}}$	(0.625, 4.375) $\rightarrow_{\succsim^{1,T}}$	(-3.625, -1.375) $\rightarrow_{\succsim^{1,F}}$	(-3.625, -1.375) $\rightarrow_{\succsim^{1,F}}$
S2	(0.625, 4.375) $\rightarrow_{\succsim^{1,T}}$	(0,0) $\rightarrow_{\succsim^{1,T}}$	(2,8) $\rightarrow_{\succsim^{1,T}}$	(-2.25, 2.25) $\rightarrow_{\succsim^{1,U}}$	(-0.75, 0.75) $\rightarrow_{\succsim^{1,U}}$
S3	(-4.375, -0.625) $\rightarrow_{\succsim^{1,F}}$	(-8, -2) $\rightarrow_{\succsim^{1,F}}$	(0,0) $\rightarrow_{\succsim^{1,T}}$	(-7.25, -2.75) $\rightarrow_{\succsim^{1,F}}$	(-7.25, -2.75) $\rightarrow_{\succsim^{1,F}}$
S4	(1.375, 3.625) $\rightarrow_{\succsim^{1,T}}$	(-2.25, 2.25) $\rightarrow_{\succsim^{1,U}}$	(2.75, 7.25) $\rightarrow_{\succsim^{1,T}}$	(0,0) $\rightarrow_{\succsim^{1,T}}$	(-1.5, 1.5) $\rightarrow_{\succsim^{1,U}}$
S5	(1.375, 3.625) $\rightarrow_{\succsim^{1,T}}$	(-0.75, 0.75) $\rightarrow_{\succsim^{1,U}}$	(2.75, 7.25) $\rightarrow_{\succsim^{1,T}}$	(-1.5, 1.5) $\rightarrow_{\succsim^{1,U}}$	(0,0) $\rightarrow_{\succsim^{1,T}}$

Table 4. Values of $m^2(S, S')$ and $M^2(S, S')$ (in parenthesis), and resulting preference relations between students for the extreme DM_2 and value function aggregation: $\succsim^{2,T}$, $\succsim^{2,F}$, and $\succsim^{2,U}$

Student	S1	S2	S3	S4	S5
S1	(0,0) $\rightarrow_{\succsim^{2,T}}$	(4.25, 5.75) $\rightarrow_{\succsim^{2,T}}$	(1,7) $\rightarrow_{\succsim^{2,T}}$	(-4.9, -3.1) $\rightarrow_{\succsim^{2,F}}$	(1.1, 2.9) $\rightarrow_{\succsim^{2,T}}$
S2	(-5.75, -4.25) $\rightarrow_{\succsim^{2,F}}$	(0,0) $\rightarrow_{\succsim^{2,T}}$	(-4.45, 2.45) $\rightarrow_{\succsim^{2,U}}$	(-10.35, -7.65) $\rightarrow_{\succsim^{2,F}}$	(-3.75, -2.25) $\rightarrow_{\succsim^{2,F}}$
S3	(-7, -1) $\rightarrow_{\succsim^{2,F}}$	(-2.45, 4.45) $\rightarrow_{\succsim^{2,U}}$	(0,0) $\rightarrow_{\succsim^{2,T}}$	(-10.7, -5.3) $\rightarrow_{\succsim^{2,F}}$	(-4.7, 0.7) $\rightarrow_{\succsim^{2,U}}$
S4	(3.1, 4.9) $\rightarrow_{\succsim^{2,T}}$	(7.65, 10.35) $\rightarrow_{\succsim^{2,U}}$	(5.3, 10.7) $\rightarrow_{\succsim^{2,T}}$	(0,0) $\rightarrow_{\succsim^{2,T}}$	(5.4, 6.6) $\rightarrow_{\succsim^{2,T}}$
S5	(-2.9, -1.1) $\rightarrow_{\succsim^{2,F}}$	(2.25, 3.75) $\rightarrow_{\succsim^{2,T}}$	(-0.7, 4.7) $\rightarrow_{\succsim^{2,U}}$	(-6.6, -5.4) $\rightarrow_{\succsim^{2,F}}$	(0,0) $\rightarrow_{\succsim^{2,T}}$

Table 5. Values of $m^3(S, S')$ and $M^3(S, S')$ (in parenthesis), and resulting preference relations between students for the moderate DM_3 and value function aggregation: $\succsim^{3,T}$, $\succsim^{3,F}$ and $\succsim^{3,U}$

Student	S1	S2	S3	S4	S5
S1	(0,0) $\rightarrow_{\succsim^{3,U}}$	(-1.5, 1.5) $\rightarrow_{\succsim^{3,U}}$	(0.75, 5.25) $\rightarrow_{\succsim^{3,T}}$	(-4.05, -1.95) $\rightarrow_{\succsim^{3,F}}$	(-2.05, 0.05) $\rightarrow_{\succsim^{3,U}}$
S2	(-1.5, 1.5) $\rightarrow_{\succsim^{3,U}}$	(0,0) $\rightarrow_{\succsim^{3,T}}$	(-15, 6.15) $\rightarrow_{\succsim^{3,U}}$	(-4.95, -1.05) $\rightarrow_{\succsim^{3,F}}$	(-1.75, -0.25) $\rightarrow_{\succsim^{3,F}}$
S3	(-5.25, -0.75) $\rightarrow_{\succsim^{3,F}}$	(-6.15, 15) $\rightarrow_{\succsim^{3,U}}$	(0,0) $\rightarrow_{\succsim^{3,T}}$	(-8.4, -3.6) $\rightarrow_{\succsim^{3,F}}$	(-6.4, -1.6) $\rightarrow_{\succsim^{3,F}}$
S4	(1.95, 4.05) $\rightarrow_{\succsim^{3,T}}$	(1.05, 4.95) $\rightarrow_{\succsim^{3,T}}$	(3.6, 8.4) $\rightarrow_{\succsim^{3,T}}$	(0,0) $\rightarrow_{\succsim^{3,T}}$	(0,0) $\rightarrow_{\succsim^{3,T}}$
S5	(-0.05, 2.05) $\rightarrow_{\succsim^{3,U}}$	(0.25, 1.75) $\rightarrow_{\succsim^{3,T}}$	(1.6, 6.4) $\rightarrow_{\succsim^{3,T}}$	(-3.2, -0.8) $\rightarrow_{\succsim^{3,F}}$	(0,0) $\rightarrow_{\succsim^{3,T}}$

Taking into account the preference relations $\succsim^{p,T}$, $\succsim^{p,F}$ and $\succsim^{p,U}$ of all decision-makers DM_p , $p = 1, 2, 3$, one can deduce in turn the overall seven-valued preference relations between students, presented in Table 6.

Table 6. Overall seven-valued preference relations between students for value function aggregation

Student	S1	S2	S3	S4	S5
S1	\succsim^T	\succsim^{fK}	\succsim^T	\succsim^F	\succsim^{fK}
S2	\succsim^{fK}	\succsim^T	\succsim^{sT}	\succsim^{sF}	\succsim^{sF}
S3	\succsim^F	\succsim^{sF}	\succsim^T	\succsim^F	\succsim^{sF}
S4	\succsim^T	\succsim^{sT}	\succsim^T	\succsim^T	\succsim^{sT}
S5	\succsim^{fK}	\succsim^{sT}	\succsim^{sT}	\succsim^{sF}	\succsim^T

3.3 Explainability of group preferences expressed in seven-valued logic

The overall seven-valued preference relations obtained for the group of decision-makers may provoke some questions concerning **explainability**, and **robustness** of results, for example, “why students $S2$ and $S3$ are in the ‘sometimes true’ preference relation”? The methodology presented so far is traceable and permits to answer such questions in the following way. The overall preference relation between $S2$ and $S3$ is ‘sometimes true’ because it is ‘true’ for the egalitarian DM_1 (Table 3), but ‘unknown’ for the extreme DM_2 (Table 4) and moderate DM_3 (Table 5). Another interesting question could be “why students $S2$ and $S1$ are in the ‘fully contradictory’ preference relation”? Remark that the preference relation between $S2$ and $S1$ is ‘true’ for the egalitarian DM_1 , ‘false’ for the extreme DM_2 , and ‘unknown’ for the moderate DM_3 . A deeper analysis is possible at the level of vertex weight vectors and their corresponding value function scores [7].

3.4 Utilization of the seven-valued preference relations in view of making a ranking recommendation

The ultimate goal of the group decision is to derive the overall ranking of students from the seven-valued preference relations among them. To achieve this, a global score $V^G(S)$ is calculated for each student S , based on how S compares to all other students, S' , using the seven-valued preference relations. The overall relations among students are presented in a graph form in Figure 3. When calculating the global score of S , a specific gain or loss value, $v(S \succsim^H S') \geq 0$, has to be assigned to each of the seven possible preference relations between S and S' , i.e., $S \succsim^H S'$, $H \in \{T, sT, U, K, fK, sF, F\}$. Similarly, a specific gain or loss value, $v(S' \succsim^H S) \geq 0$, has to be assigned to each of the seven possible preference relations between S' and S , i.e., $S' \succsim^H S$, $H \in \{T, sT, U, K, fK, sF, F\}$. Consequently, the global score of student S is calculated as:

$$\begin{aligned}
 V^G(S) = & \sum_{\forall S' \neq S} \sum_{H \in \{T, sT\}} v(S \succsim^H S') - \sum_{\forall S' \neq S} \sum_{H \in \{sF, F\}} v(S \succsim^H S') \\
 & - \sum_{\forall S' \neq S} \sum_{H \in \{T, sT\}} v(S' \succsim^H S) + \sum_{\forall S' \neq S} \sum_{H \in \{sF, F\}} v(S' \succsim^H S).
 \end{aligned}$$

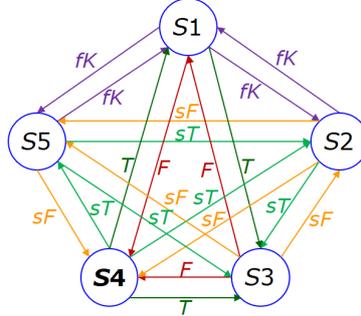


Fig. 3. The graph of the overall seven-valued preference relations in the set of students.

The values assigned to the gains or losses, $v(S \succsim^H S')$ and $v(S' \succsim^H S)$, $H \in \{T, sT, U, K, fK, sF, F\}$, have to respect the conditions specified in [7]. The following simple convention of assigning values to gains and losses satisfies these conditions:

- $v(S \succsim^T S') = v(S' \succsim^F S) = 1$,
- $v(S \succsim^{sT} S') = v(S' \succsim^{sF} S) = 0.5$,
- $v(S \succsim^U S') = v(S \succsim^K S') = v(S \succsim^{fK} S') = 0$,
- as well as $v(S' \succsim^U S) = v(S' \succsim^K S) = v(S' \succsim^{fK} S) = 0$,
- $v(S \succsim^{sF} S') = v(S' \succsim^{sT} S) = 0.5$,
- $v(S \succsim^F S') = v(S' \succsim^T S) = 1$.

In doing so, the global scores obtained by students is as follows:

$$V^G(S1) = 0, V^G(S2) = -1, V^G(S3) = -6, V^G(S4) = 6, V^G(S5) = 1.$$

Thus, the group ranking of students according to the above way of utilization of the overall seven-valued preference relations is: $S4 \rightarrow S5 \rightarrow S1 \rightarrow S2 \rightarrow S3$.

3.5 Conclusions

We presented a basic version of the decision aiding methodology for group decision in the multi-criteria context, based on seven-valued logic. Although the example illustrating the methodology involves only three decision-makers, the methodology can accommodate a larger number. Moreover, when the decision-makers are experts who assign different performance levels to the alternatives on the considered criteria, the methodology still arrives at a compromise group recommendation, as the true, false, and unknown preference relations are calculated in parallel for these experts and their performance matrices. As shown or announced in [7], other generalizations are possible. In particular,

- Another weighted aggregation of multi-criteria evaluations can be used, e.g., the Choquet integral [4], or an outranking function known from ELECTRE-like methods [2]. In the latter case, the weights do not specify trade-offs,

as they do in value functions, but instead indicate the relative strengths of criteria, as in a voting procedure. The ‘deck of the cards’ method described in [3] is appropriate for this task.

- To determine values of gains and losses $v(S \succsim^H S')$, $v(S' \succsim^H S)$, $H \in \{T, sT, U, K, fK, sF, F\}$ in the scoring function, one can also use the ‘deck of cards’ method, instead of the simple convention used in our example.
- To avoid bias resulting from considering the weighted aggregation with weight vectors located at the vertices of $E_{(wp)}^p$ only, can use SMAA (Stochastic Multicriteria Acceptability Analysis) [9, 10] with a uniform probability distribution in the space of feasible weights.
- Instead of perturbation of central weight vectors, one can obtain a cloud of feasible weights in result of Robust Ordinal Regression [6, 8] on indirect preference information having the form of holistic pairwise comparisons of some reference alternatives by the decision-makers.

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